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A THREE-DIMENSIONAL STEADY-STATE ATMOSPHERIC DISPERSION–DEPOSITION MODEL FOR EMISSIONS FROM A GROUND-LEVEL AREA SOURCE

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Abstract—An analytical solution to the steady-state three-dimensional atmospheric dispersion equation has been developed for the transport of non-buoyant emissions from a continuous ground-level area source. The model incorporates power law profiles for the variation of wind speed and vertical eddy diffusivity with height, represents the lateral eddy diffusivity as a function of wind speed and the crosswind dispersion coefficient, and includes dry deposition as a removal mechanism. The model is well suited for accurate prediction of emission concentration levels in the vicinity of an area source, as well as farther downwind, under neutral or stable atmospheric conditions. The impact of the important model parameters on contaminant dispersion is examined. The results from several simulations, compared with point and line sources of equivalent source strength, indicate that at short downwind distances, predictions of contaminant concentrations emitted from area sources may be unacceptably inaccurate unless the structure of the source is properly taken into account.

Key word index: Atmospheric dispersion, dry deposition, area source, non-buoyant emissions, mathematical modeling.

NOMENCLATURE

a	parameter used in the power law wind profile, dependent on q_{z_0} and atmospheric conditions $(L^{1-m}T^{-1})$
A	defined in Equation (21)
b	parameter used in power law vertical eddy diffusivity profile, dependent on q_{z_0} and atmospheric conditions $(L^{2-n}T^{-1})$
C	contaminant concentration (ML^{-3})
C'	fluctuating component of concentration (ML^{-3})
$\langle C \rangle$	ensemble average component of concentration (ML^{-3})
D	molecular diffusion coefficient (L^2T^{-1})
erf	error function
$E_{a,b}(z)$	generalized Mittag–Leffler function
F_d	dry deposition flux $(ML^{-2}T^{-1})$
$\mathbf{F}$	arbitrary function
J_a	rate of mass emitted per unit area $(ML^{-2}T^{-1})$
J_l	rate of mass emitted per unit length $(ML^{-1}T^{-1})$
J_p	rate of total mass emitted (MT^{-1})
k	integer
K_{ij}	eddy diffusivities (L^2T^{-1})
$\mathbf{K}$	eddy diffusivity second-order tensor
l_x	downwind length of the source (L)
l_y	crosswind width of the source (L)
L	Monin–Obukhov stability length (L)
m	parameter used in power law wind profile, dependent on atmospheric conditions and ground surface roughness
M	confluent hypergeometric function, defined in Equation (18b)
n	parameter used in power law vertical eddy diffusivity profile, dependent on atmospheric conditions and ground surface roughness
O	order of magnitude
q_i	cartesian coordinate axes (L)
q_{z_0}	surface roughness length (L)
q_z^*	some reference height (L)
$\mathbf{q}$	vector of spatial coordinates
S	source function $(ML^{-3}T^{-1})$
t	time (T)
$\bar{u}_i$	deterministic mean fluid velocity components (LT^{-1})
u_i'	fluctuating or stochastic fluid velocity components (LT^{-1})
$\bar{\mathbf{u}}$	deterministic mean fluid velocity vector
$\mathbf{u}'$	fluctuating or stochastic fluid velocity vector
U	Kummer's hypergeometric function
v_d	dry deposition velocity (LT^{-1})
β	defined in Equation (16b), dimensionless
γ	defined in Equation (16c), dimensionless
Γ	gamma function
δ	Dirac delta function
ζ	constant used in the definition of σ_y^2, depends on atmospheric conditions $(L^{\eta/2})$
η	constant used in the definition of σ_y^2, depends on atmospheric conditions, dimensionless
Θ	defined in Equation (25), dimensionless
Λ	defined in Equation (16f) $(L^{-\nu})$
ν	defined in Equation (16d), dimensionless
ξ, ξ_0	defined in Equations (34) and (35), respectively, dimensionless
σ_y^2	crosswind mean square particle displacement (L^2)
Φ_m	defined in Equation (32), dimensionless
Ψ	defined in Equation (33), dimensionless
Ω	defined in Equation (16e), dimensionless
$\nabla_{\mathbf{q}}$	vector differential operator
$\nabla_{\mathbf{q}}^2$	three-dimensional cartesian Laplacian operator
$\vert\vert$	absolute value

Subscripts

i, j	direction of principal axes: $i, j = x, y, z$
x, y, z	principal directions of a cartesian coordinate system

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Superscripts

- T transpose
- ° implies contaminant source location
- ‾ (overbar) average over time and space.

INTRODUCTION

Emissions of volatile organic contaminants from hazardous waste landfills, municipal wastewater and contaminated groundwater treatment facilities can create potential health risks to on-site workers and to the general public in surrounding communities (Hwang, 1985; Dunovant *et al.*, 1986; Namkung and Rittmann, 1987). To evaluate the potential hazard on the local atmosphere, accurate determination of volatile organic contaminant emission rates and modeling of atmospheric dispersion is necessary. The Gaussian plume model for turbulent atmospheric dispersion of a contaminant, despite a popularity and wide use (Baker and MacKay, 1985; Hinrichsen, 1986; Wolfinger, 1989) which is attributed to the simplicity of its formulation, is not adequate for ground-level sources because the mean wind velocity is assumed to be uniform and the vertical eddy diffusivity constant. Hence Gaussian plume models are inappropriate for simulation of contaminant dispersion in the boundary layer near a ground-level source.

For a realistic description of turbulent dispersion near the surface of the earth, it is essential to account for the variation of the mean wind velocity and the vertical eddy diffusivity with height above the ground. Several theoretical and empirical expressions are available for the mean velocity and the eddy diffusivities as functions of the coordinates (see, e.g. Panofsky, 1961; Swinbank, 1968; O'Brien, 1970; Monin and Yaglom, 1971; Businger and Arya, 1974; Lamb *et al.*, 1975; Crane *et al.*, 1977; Lamb and Duran, 1978). The commonly-used power law approximations for the profiles of wind velocity and vertical diffusivity have been employed to derive analytical solutions to several atmospheric dispersion models (Smith, 1957; Huang, 1979; Rao, 1981; Koch, 1989; Chitgopekar *et al.*, 1990), and have been applied successfully to the prediction of evaporation (Brutsaert and Yeh, 1970) and atmospheric dispersion from instantaneous sources (Drivas and Shair, 1974; Dvore and Vaglio-Laurin, 1982).

The major sink mechanisms of a non-reactive atmospheric contaminant are rainout and dry deposition (Bolin *et al.*, 1974). Dry deposition is important for removing airborne contaminants at the Earth's surface layer, while rainout is a significant sink mechanism at greater heights. Rainout implies contaminant removal during cloud formation or contaminant sorption to cloud elements, and is not accounted for in this work. Dry deposition is a complicated process unrelated to precipitation, and represents the impingement or sorption of a contaminant onto the Earth's surface. Some of the micrometeorological factors influencing dry deposition removal rates are aerodynamic roughness, atmospheric stability, contaminant concentration, relative humidity, seasonal variation, solar radiation, temperature, turbulence, and wind velocity (Sehmel, 1980). Several available models for atmospheric contaminant transport include dry deposition as a sink mechanism (see, e.g. Scriven and Fisher, 1975; Shreffler, 1975; Overcamp, 1976; Slinn, 1977; Draxler and Elliot, 1977; Horst, 1977; Rao, 1981; Koch, 1989), but do not account both for a ground-level area source and for the variation of wind speed and eddy diffusivities with height.

The present work focuses on the atmospheric dispersion of non-reacting volatile organic contaminants emitted from ground-level sources such as hazardous waste sites and publicly owned treatment works. Analytical procedures are employed to solve the three-dimensional steady-state atmospheric dispersion equation with spatially-variable wind velocity and eddy diffusivities. Inclusion of dry deposition in this three-dimensional area-source model as a potential sink for the ground-level emissions represents an important step towards more accurately predicting concentrations. Mathematical methods for estimation of the model parameters are also presented, including a rapidly-converging approximation for evaluating a component of the analytical solution known as the Kummer hypergeometric function.

ATMOSPHERIC DISPERSION MODEL

The transport of a single inert contaminant released into an atmosphere possessing highly irregular and chaotic turbulent motion is governed by the following stochastic partial differential equation (Seinfeld, 1986, p. 524)

$$\frac{\partial\langle C(t,\mathbf{q})\rangle}{\partial t}+\nabla_{\mathbf{q}}\cdot[\bar{\mathbf{u}}(t,\mathbf{q})\langle C(t,\mathbf{q})\rangle]+\nabla_{\mathbf{q}}\cdot\langle \mathbf{u}'(t,\mathbf{q})C'(t,\mathbf{q})\rangle=D\nabla_{\mathbf{q}}^2\langle C(t,\mathbf{q})\rangle+S(t,\mathbf{q}^0),\qquad(1)$$

where $C=\langle C\rangle+C'$ is the contaminant concentration; $\langle C\rangle$ is the ensemble average concentration; C' is the concentration fluctuation ($\langle C'\rangle=0$); $\mathbf{q}=(q_x,q_y,q_z)^{\mathrm{T}}$ is the vector of spatial coordinates, and subscripts x, y, z denote the principal directions of a cartesian coordinate system; $\bar{\mathbf{u}}=(\bar{u}_x,\bar{u}_y,\bar{u}_z)^{\mathrm{T}}$ is the deterministic mean fluid velocity vector; $\mathbf{u}'=(u'_x,u'_y,u'_z)^{\mathrm{T}}$ is the fluctuating or stochastic fluid velocity vector ($\langle\mathbf{u}'\rangle=0$); D is the molecular diffusion coefficient of the contaminant; S is a source function located at $\mathbf{q}^0=(q_x^0,q_y^0,q_z^0)^{\mathrm{T}}$; $\nabla_{\mathbf{q}}$ is the vector differential operator ($\nabla_{\mathbf{q}}=[\partial/\partial q_x,\partial/\partial q_y,\partial/\partial q_z]^{\mathrm{T}}$); $\nabla_{\mathbf{q}}\cdot$ denotes divergence ($\nabla_{\mathbf{q}}\cdot\mathbf{F}=\partial F_x/\partial q_x+\partial F_y/\partial q_y+\partial F_z/\partial q_z$); $\nabla_{\mathbf{q}}^2$ is the Laplacian operator ($\nabla_{\mathbf{q}}^2\mathbf{F}=\nabla_{\mathbf{q}}\cdot\nabla_{\mathbf{q}}\mathbf{F}$); and $\mathbf{F}$ is an arbitrary function. Ordinarily, if the atmospheric flow is not buoyancy driven and the transport mechanism length scale is much smaller than the distance over which the mean transported field gradient changes significantly, the flux $\langle \mathbf{u}'C' \rangle$ can be related to $\langle C \rangle$ by (Monin and Yaglom, 1971)

$$\langle \mathbf{u}'(t, \mathbf{q}) C'(t, \mathbf{q}) \rangle = -\mathbf{K}(t, \mathbf{q}) \cdot \nabla_{\mathbf{q}} \langle C(t, \mathbf{q}) \rangle, \quad (2)$$

where

$$\mathbf{K} = \begin{pmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{pmatrix} \quad (3)$$

is the eddy diffusivity second-order tensor. $\mathbf{K}$ is a diagonal matrix only for a cartesian coordinate system coinciding with the principal axes of the eddy diffusivity tensor. (Corrsin, 1974, showed how to proceed in situations where the off-diagonal terms are retained.) Combining Equations (1) and (2), and assuming that the molecular diffusion term, $D\nabla_{\mathbf{q}}^2 \langle C \rangle$, is negligible compared with the turbulent diffusion term, $\nabla_{\mathbf{q}} \cdot \langle \mathbf{u}'C' \rangle$, and that the density of the fluid remains constant following the motion so that the fluid is incompressible, i.e. the velocity vector is non-divergent ($\nabla_{\mathbf{q}} \cdot \bar{\mathbf{u}} = 0$), leads to

$$\frac{\partial \langle C(t, \mathbf{q}) \rangle}{\partial t} + \bar{\mathbf{u}}(t, \mathbf{q}) \cdot \nabla_{\mathbf{q}} < C(t, \mathbf{q}) \rangle = \nabla_{\mathbf{q}} \cdot [\mathbf{K}(t, \mathbf{q}) \cdot \nabla_{\mathbf{q}} < C(t, \mathbf{q}) \rangle] + S(t, \mathbf{q}^0). \quad (4)$$

For steady unidirectional flow along the q_x coordinate over a flat terrain and steady contaminant emission rate, $\bar{u}_y$, $\bar{u}_z$, and $\partial \langle C \rangle / \partial t$ are equal to zero. Furthermore, assuming that the transport in the x-direction due to the wind is dominant over turbulent dispersion in the x-direction (slender plume approximation), from (4) we obtain the equation

$$\bar{u}_x(\mathbf{q}) \frac{\partial \langle C(\mathbf{q}) \rangle}{\partial q_x} = \frac{\partial}{\partial q_y} \left(K_{yy}(\mathbf{q}) \frac{\partial \langle C(\mathbf{q}) \rangle}{\partial q_y} \right) + \frac{\partial}{\partial q_z} \left(K_{zz}(\mathbf{q}) \frac{\partial \langle C(\mathbf{q}) \rangle}{\partial q_z} \right) + S(\mathbf{q}^0), \quad (5)$$

where

$$S(\mathbf{q}^0) = J_p \delta(\mathbf{q} - \mathbf{q}^0), \quad (6)$$

J_p is the rate of total mass emitted, and

$$\delta(\mathbf{q} - \mathbf{q}^0) = \delta(q_x - q_x^0)\delta(q_y - q_y^0)\delta(q_z - q_z^0) = \begin{cases} 0 & \mathbf{q} \neq \mathbf{q}^0, \\ 1 & \mathbf{q} = \mathbf{q}^0 \end{cases}. \quad (7)$$

For mathematical convenience, $\bar{u}_x$ and K_{zz} are approximated by the following power functions of q_z, respectively,

$$\bar{u}_x(q_z) = a q_z^m, \quad (8)$$

$$K_{zz}(q_z) = b q_z^n, \quad (9)$$

where the parameters a, b, m, and n are not constants, but depend on the atmospheric conditions and on the ground surface roughness. Furthermore, the lateral eddy diffusivity is represented by the following expression (Huang, 1979)

$$K_{yy}(q_x, q_z) = \frac{1}{2} \bar{u}_x(q_z) \frac{d\sigma_y^2(q_x)}{dq_x}, \quad (10)$$

where σ_y^2 is the mean square displacement along the q_y coordinate axis (crosswind) of a fluid particle released from the source. Substituting (8)–(10) into the governing Equation (5) leads to

$$a q_z^m \frac{\partial \langle C(\mathbf{q}) \rangle}{\partial q_x} = \frac{a q_z^m}{2} \frac{d\sigma_y^2(q_x)}{dq_x} \frac{\partial^2 \langle C(\mathbf{q}) \rangle}{\partial q_y^2} + b q_z^n \frac{\partial^2 \langle C(\mathbf{q}) \rangle}{\partial q_z^2} + n b q_z^{n-1} \frac{\partial \langle C(\mathbf{q}) \rangle}{\partial q_z} + S(\mathbf{q}^0). \quad (11)$$

Since K_{yy} is independent of q_y, the atmospheric diffusion Equation (11) accounts only for simple diffusion in the lateral direction. Assuming that the contaminant is initially absent from the atmosphere and the only sink mechanism is dry deposition, the appropriate boundary conditions are

$$\langle C(\infty, q_y, q_z) \rangle = 0, \quad (12)$$

$$\langle C(q_x, \pm\infty, q_z) \rangle = 0, \quad (13)$$

$$\langle C(q_x, q_y, \infty) \rangle = 0, \quad (14)$$

$$K_{zz}(q_z) \frac{\partial \langle C(\mathbf{q}) \rangle}{\partial q_z} = v_d \langle C(\mathbf{q}) \rangle \quad \text{at} \quad q_z = q_{z_0}, \quad (15)$$

where v_d is the deposition velocity, and q_{z_0} is the surface roughness length. Boundary condition (15) indicates that the turbulent transport of the contaminant along the vertical concentration gradient is balanced by the net contaminant flux to the Earth's surface resulting from an exchange between the atmosphere and the Earth's surface.

Following the procedures of Yih (1952), Yeh (1975) and Koch (1989), the solution of Equations (11)–(15) for a ground-level point source located at $\mathbf{q}^0 = (q_x^0, q_y^0, 0)^T$ can be obtained as

$$\langle C(\mathbf{q}) \rangle = \frac{J_p \beta \Omega}{a^\nu \sqrt{2\pi[\sigma_y^2(q_x) - \sigma_y^2(q_x^0)]}\, [\beta^2 b(q_x - q_x^0)]^{1-\nu}\, \Gamma(1-\nu)} \times \exp\left[-\frac{(q_y - q_y^0)^2}{2\{\sigma_y^2(q_x) - \sigma_y^2(q_x^0)\}} - \gamma \right] \qquad (q_x > q_x^0), \quad (16a)$$

where

$$\beta = m - n + 2, \tag{16b}$$

$$\gamma = \frac{a q_z^{\beta}}{\beta^2 b (q_x - q_x^0)}, \tag{16c}$$

$$\nu = \frac{1-n}{\beta}, \tag{16d}$$

$$\Omega = 1 + \sum_{k=1}^{\infty} [-\Lambda (q_x - q_x^0)^{\nu}]^k \, \mathrm{U}(\nu k, 1-\nu, \gamma), \tag{16e}$$

$$\Lambda = -\frac{v_d}{\nu b}\left(\frac{b}{a}\right)^{\nu} \frac{\Gamma(1+\nu)}{\Gamma(1-\nu)} \beta^{2\nu-1}, \tag{16f}$$

$\Gamma(z)$ is the gamma function and $\mathrm{U}(a, b, z)$ is the Kummer hypergeometric function. The gamma function is defined by the integral (Davis, 1972)

$$\Gamma(z) = \int_0^{\infty} t^{z-1} \, \mathrm{e}^{-t} \, \mathrm{d}t, \qquad (z > 0) \tag{17}$$

and can be calculated numerically by the approximation derived by Lanczos (1964). The Kummer function is defined by (Slater, 1972)

$$\mathrm{U}(a, b, z) = \frac{\pi}{\sin \pi b} \left\{ \frac{M(a, b, z)}{\Gamma(1+a-b)\Gamma(b)} - z^{1-b} \frac{M(1+a-b, 2-b, z)}{\Gamma(a)\Gamma(2-b)} \right\} \tag{18a}$$

where

$$M(a, b, z) = 1 + \sum_{k=1}^{\infty} \frac{a(a+1)\ldots(a+k-1)z^k}{b(b+1)\ldots(b+k-1)k!}. \tag{18b}$$

For small heights at sufficient distances from the contaminant source ($q_z \ll q_x$), $\gamma \to 0$ and the Kummer function reduces to (Slater, 1972)

$$\mathrm{U}(\nu k, 1-\nu, \gamma) = \frac{\Gamma(\nu)}{\Gamma(\nu + \nu k)} \qquad |\gamma| \to 0. \tag{19}$$

In view of Equation (19), Ω can be written as

$$\Omega = 1 + \Gamma(\nu) \sum_{k=1}^{\infty} \frac{A^k}{\Gamma(\nu + \nu k)} = \Gamma(\nu) E_{\nu,\nu}(A), \tag{20}$$

where

$$A = -\Lambda (q_x - q_x^0)^{\nu}, \tag{21}$$

and $E_{a,b}(A)$ is a generalized Mittag–Leffler function defined as (Erdélyi *et al.*, 1955; Marichev, 1983)

$$E_{a,b}(A) = \sum_{k=0}^{\infty} \frac{A^k}{\Gamma(b + ak)} \qquad a, b > 0. \tag{22}$$

For large A, the Mittag–Leffler type function becomes (Erdélyi *et al.*, 1955, p. 210)

$$E_{\nu,\nu}(A) = -\sum_{k=1}^{K-1} \frac{A^{-k}}{\Gamma(\nu - \nu k)} + O(|A|^{-K}) \qquad |A| \to \infty. \tag{23}$$

The preceding equation is applicable whenever the approximation to the Kummer function (19) is valid, because $|\gamma| \to 0$ implies that $q_x \to \infty$ and thus $|A| \to \infty$. Even though the series in the confluent hypergeometric function (Equation (18b)) and in the generalized Mittag–Leffler function (Equation (22)) may be convergent, the intermediate terms can become quite large, necessitating that overflow conditions be checked continually when performing numerical evaluations. However, Equation (23) converges fast without numerical overflow.

Crosswind continuous line source

Integrating Equation (16a) for a ground-level crosswind continuous line source of finite length l_y (this corresponds to an integration from zero to l_y with respect to q_y^0), we obtain

$$\langle C(\mathbf{q}) \rangle = \frac{J_l \beta \Theta \Omega \exp[-\gamma]}{2a^{\nu}[\beta^2 b (q_x - q_x^0)]^{1-\nu} \Gamma(1-\nu)} \qquad (q_x > q_x^0), \tag{24}$$

where

$$\Theta = erf\left[\frac{l_y/2 + q_y}{\sqrt{2\{\sigma_y^2(q_x) - \sigma_y^2(q_x^0)\}}}\right] + erf\left[\frac{l_y/2 - q_y}{\sqrt{2\{\sigma_y^2(q_x) - \sigma_y^2(q_x^0)\}}}\right], \tag{25}$$

$$erf[z] = \frac{2}{\sqrt{\pi}} \int_0^z \exp[-x^2] \, \mathrm{d}x, \tag{26}$$

and J_l is the rate of mass emitted per unit length. Similarly, integrating Equation (16a) for a ground-level infinite crosswind continuous line source and setting $q_x^0 = 0$, we obtain

$$\langle C(q_x, q_z) \rangle = \frac{J_l \beta \Omega \exp[-\gamma]}{a^{\nu}(\beta^2 b q_x)^{1-\nu} \Gamma(1-\nu)}. \tag{27}$$

The preceding equation was derived by Koch (1989).

Zero dry deposition velocity

For the special case where v_d equals zero, $\Lambda = 0$, $\Omega = 1$, and consequently for $q_x^0 = 0$, Equation (16a) reduces to

$$\langle C(\mathbf{q}) \rangle = \frac{J_p \beta}{a^{\nu} \sqrt{2\pi}\, \sigma_y(q_x)(\beta^2 b q_x)^{1-\nu} \Gamma(1-\nu)} \exp\left[-\frac{(q_y - q_y^0)^2}{2\sigma_y^2(q_x)} - \gamma\right]. \tag{28}$$

Equation (28) was derived by Huang (1979).

Ground-level finite area source

For a ground-level area source of downwind length l_x and crosswind width l_y, Equation (16a) can be integrated to yield

$$\langle C(\mathbf{q}) \rangle = \int_0^{l_x} \frac{J_a \beta \Theta \Omega \exp[-\gamma]}{2a^{\nu}[\beta^2 b (q_x - q_x^0)]^{1-\nu} \Gamma(1-\nu)} \, \mathrm{d}q_x^0 \qquad (q_x > l_x), \tag{29}$$

where J_a is the rate of mass emitted per unit area. Since analytical evaluation of the integral in the preceding equation is not straightforward, numerical integration techniques must be employed.

EVALUATION OF ATMOSPHERIC DISPERSION MODEL PARAMETERS

The parameters for the atmospheric dispersion model previously described can be estimated by the following relationships. The information required is the wind speed and vertical eddy diffusivity at a reference height, and appropriate classification of the atmospheric conditions.

The multiplication constant, a, and the exponent, m, in the power law wind profile can be obtained either from experimental wind speed measurements or can be determined analytically on the basis of the Monin–Obukhov similarity theory which leads to the following expressions (Dyer and Hicks, 1970; Paulson, 1970; Webb, 1970; Huang and Nickerson, 1972; Dyer, 1974)

$$a=\frac{\bar{u}_x(q_z^*)}{(q_z^*)^m} \tag{30}$$

$$m=\frac{\Phi_m\left(\dfrac{q_z}{L}\right)}{\Psi\left(\dfrac{q_z}{L},\dfrac{q_{z0}}{L}\right)} \qquad q_z>q_{z_0}, \tag{31}$$

where

$$\Phi_m\left(\frac{q_z}{L}\right)=\begin{cases}(1-16q_z/L)^{-0.25} & q_z/L<0,\\ 1+5q_z/L & q_z/L\geqslant 0,\end{cases} \tag{32}$$

$$\Psi\left(\frac{q_z}{L},\frac{q_{z0}}{L}\right)=\begin{cases}\ln\left[\dfrac{(\xi-1)(\xi_0+1)}{(\xi+1)(\xi_0-1)}\right]+2(\tan^{-1}\xi-\tan^{-1}\xi_0) & q_z/L<0,\\ \ln\left[\dfrac{q_z}{q_{z0}}\right]+5\dfrac{q_z}{L} & q_z/L\geqslant 0,\end{cases} \tag{33}$$

$$\xi=\Phi_m^{-1}\left(\frac{q_z}{L}\right), \tag{34}$$

$$\xi_0=\Phi_m^{-1}\left(\frac{q_{z0}}{L}\right), \tag{35}$$

q_z^* is a reference height; $\bar{u}_x(q_z^*)$, if unknown, can be evaluated by the approximations derived by Benoit (1977); and L is the Monin–Obukhov stability length, which refers to the height above the Earth's surface where the contribution to the production of turbulence by both mechanical and buoyancy forces is equal. Note that $q_z/L<0$ and $q_z/L\geqslant 0$ refer to unstable and neutral/stable conditions, respectively. Since L is not easily measured experimentally, it can be approximated by available correlations {see Seinfeld (1986, p. 511)} of the relations between stability parameters in the surface layer established by Golder (1972).

The multiplication constant, b, and the exponent, n, in the power law expression of the vertical eddy diffusivity can be determined analytically on the basis of the Monin–Obukhov similarity theory as (Huang, 1979)

$$b=\frac{K_{zz}(q_z^*)}{(q_z^*)^n}, \tag{36}$$

$$n=\begin{cases}\dfrac{1-20q_z/L}{1-16q_z/L} & q_z/L<0,\\ 1 & q_z/L=0,\\ (1+5q_z/L)^{-1} & q_z/L>0.\end{cases} \tag{37}$$

The crosswind mean square particle displacement is commonly treated as an empirical dispersion coefficient which can be determined by fitting experimental data. Several functional forms for the variance σ_y^2 have been proposed (e.g. Deardorff and Willis, 1975; Draxler, 1976; Willis and Deardorff, 1976; Panofsky *et al.*, 1977; Nieuwstadt, 1980); however, only the relationship developed by Sutton (1932) as shown by Huang (1979) is presented

$$\sigma_y^2(q_x)=\frac{1}{2}\zeta^2 q_x^{2-\eta}, \tag{38}$$

$$\eta=\frac{2m}{1+m}, \tag{39}$$

Table 1. Relationship between ζ, η and atmospheric stability classes*

Stability class	Pasquill (1961)	ζ	η
Extremely unstable	A	0.56	0.19
Moderately unstable	B	0.50	0.28
Slightly unstable	C	0.50	0.28
Neutral	D	0.45	0.45
Slightly stable	E		
Moderately stable	F	0.44	0.58

* Modified tabulation from Huang (1979).

where the parameters ζ and η depend on the atmospheric conditions, as shown in Table 1.

The dry deposition velocity is defined by Chamberlain and Chadwick (1953) as the gas or particle deposition flux, F_d, divided by an airborne concentration

$$v_d = \frac{F_d}{\langle C \rangle}, \tag{40}$$

where all three quantities generally vary with height and time. It should be pointed out that F_d refers to the amount of gas or particle deposition per unit time per unit area of ground plan and not per unit area of actual surface (McMahon and Denison, 1979). The dry deposition velocity is weakly dependent on height if F_d is assumed constant in the lowest atmospheric layer (Slinn, 1978). In this study the preceding equation is assumed to be time invariant and is used only as a boundary condition at $q_z = 0$. Theoretical calculations and experimental measurements of dry deposition for organic pollutants (i.e. polycyclic aromatic hydrocarbons, polychlorinated biphenyls, pesticides) show values of v_d ranging between 0.05 and 1.0 cm s^{-1} (Bidleman, 1988) under typical conditions.

DISCUSSION

The variation of the mean wind velocity and eddy diffusivities with height is illustrated in Figs 1a–1c, for the Pasquill stability classes listed in Table 1, assuming that $q_{z_0} = 0.1$ m, $\bar{u}_x(q_z^*) = 1.5$ m s^{-1} and $K_{zz}(q_z^*) = 0.025$ m^2 s^{-1} at a reference height $q_z^* = 10$ m. The multiplication constant a, and the exponent m, in the power law wind expression (8) are obtained by Equations (30) and (31), respectively. Similarly, b and n are obtained by Equations (36) and (37), respectively. Furthermore, the crosswind mean square particle displacement $\sigma_y^2(q_x)$, in the lateral eddy diffusivity expression (10) is obtained by Equation (38). Figures 1a–1c illustrate the significant increase in the mean wind velocity and eddy diffusivities with height above the ground, and also indicate the importance of accurate determination of atmospheric conditions, since the shape of the profiles is drastically affected by the different stability classes. It should be noted that the power laws (8) and (9) are good approximations of the wind and vertical eddy diffusivity profiles under neutral and stable conditions, but in the case of unstable atmospheric conditions they only adequately represent the real profiles very close to the ground.

To illustrate the expected behavior of the three-dimensional atmospheric dispersion model for ground-level area sources, concentration profiles have been calculated for a variety of conditions. The curves appearing in Fig. 2 and subsequent figures are obtained from Equation (29) at moderately stable atmospheric conditions, for a ground-level area source of crosswind width and downwind length 40 and 25 m, respectively. The procedures and associated assumptions previously described are used for the determination of a, b, m, n, and σ_y^2. The appropriate Monin–Obukhov stability length is obtained from Table 2. The gamma functions are calculated numerically by the approximation derived by Lanczos (1964), and the integral in (29) is evaluated numerically by the extended Simpson's rule (Press et al., 1986). For presentation purposes, the calculated concentrations are normalized by the rate of mass emitted per unit area.

Concentration profiles along the three Cartesian coordinate axes are shown in Fig. 2. The concentration along the centerline of the plume in the downwind direction at a given elevation reaches a point of

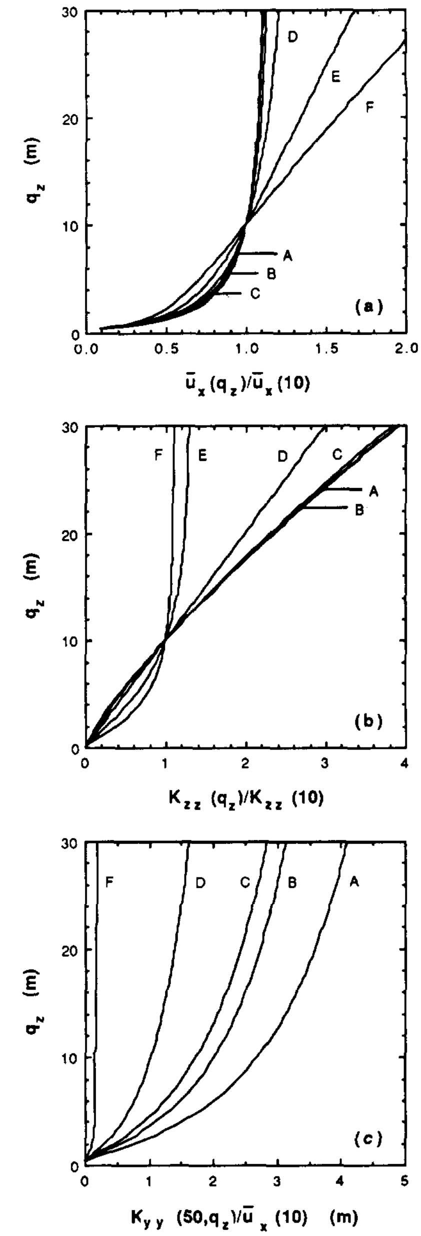

Fig. 1. Normalized (a) wind, (b) vertical eddy diffusivity, and (c) lateral eddy diffusivity profiles for various Pasquill stability conditions.

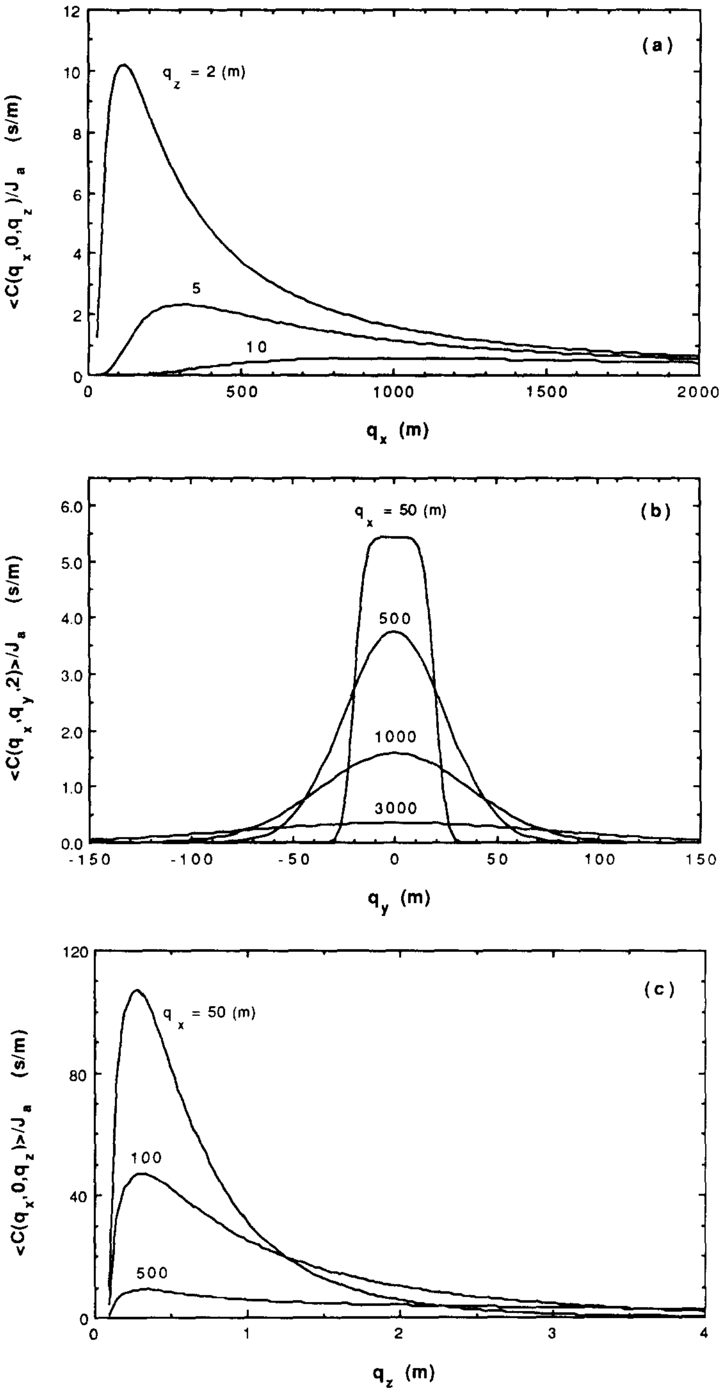

Fig. 2. Distribution of normalized concentration vs (a) downwind distance for $q_z=2$, 5, 10 m, (b) crosswind distance for $q_x=50$, 500, 1000, 3000 m, and (c) vertical distance for $q_x=50$, 100, 250, 1000 m, (F Stability, $q_{z_0}=0.1$ m, $v_d=0$).

maximum concentration followed by an extended tailing. The position of the point of maximum concentration shifts away from the source with increasing vertical distance (Fig. 2a). In the crosswind direction, the concentration profiles are symmetric with peak concentrations along the centerline of the plume. Concentration levels decrease and lateral spreading increases with increasing downwind distance from the source (Fig. 2b). In the vertical direction, the concentration profiles at the plume centerline are found to depend on the variation of wind velocity and vertical eddy diffusivity. As the downwind distance increases the vertical concentration profiles approach uniform distribution due to the vertical mixing (Fig. 2c).

Table 2. Values of $L^{-1}(m^{-1})$ used for the different atmospheric stability conditions*

	q_{z_0}(m)			
Stability class	0.01	0.1	1.0	2.0
Extremely unstable	−0.154	−0.125	−0.096	−0.087
Moderately unstable	−0.095	−0.066	−0.037	−0.028
Slightly unstable	−0.038	−0.020	−0.002	−0.003
Neutral	0.000	0.000	0.000	0.000
Slightly stable	0.040	0.022	0.004	0.001
Moderately stable	0.107	0.071	0.035	0.024

* From Golder (1972) and Myrup and Ranzieri (1976).

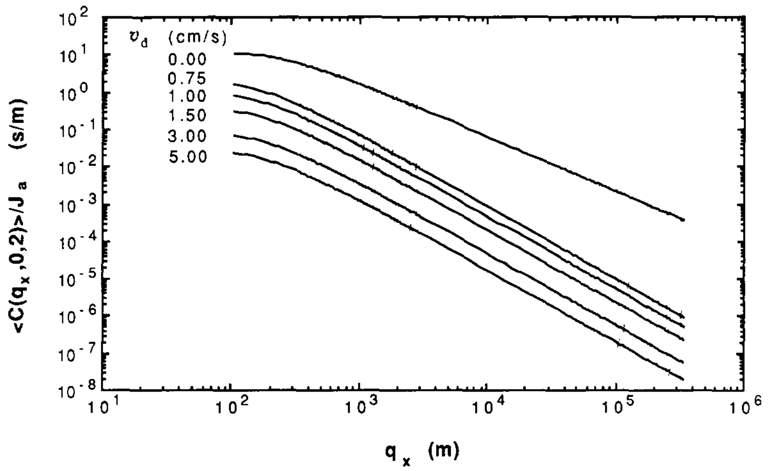

Fig. 3. Variation of normalized concentration with downwind distance and dry deposition velocity for $q_y=0$ m, $q_z=2$ m and moderately stable atmospheric conditions.

In Fig. 3, we have plotted concentration profiles for dry deposition velocities in the range of 0.0–5.0 cm s^{-1}. The parameter Ω is estimated with Equations (20) and (23), assuming that $q_z \ll q_x$ so that the approximation to the Kummer function (Equation (19)) is valid. It should be noted that since the Mittag–Leffler type function (23) can be used easily, without numerical overflow problems, this approach greatly facilitates utilization of both this three-dimensional model and the two-dimensional model derived by Koch (1989). Figure 3 indicates that with increasing v_d, the concentration reduces significantly.

Figure 4 shows normalized concentration profiles along the centerline at a vertical height of 2 m for the atmospheric stability classes presented in Table 1. The variation of the Monin–Obukhov stability length with atmospheric stability and surface roughness is obtained from Table 2. For unstable conditions the peak concentration near ground-level is reduced owing to the increased turbulence (Fig. 4).

The effect of surface roughness on the concentration distribution at moderately stable atmospheric stability is shown in Fig. 5. Typical values of the surface roughness are 0.01 m for lawn, 0.1 m for fully grown root crops, 1 m for tree covered areas, and 2 m for low-density residential districts (McRae *et al.*, 1982). For these surface roughness lengths, the appropriate Monin–Obukhov lengths as a function of atmospheric stability conditions (Golder, 1972; Myrup and Ranzieri, 1976), have been listed in Table 2. As the surface roughness is increased, the resistance to downwind contaminant transport increases, leading to increased peak concentrations (Fig. 5).

Figure 6 shows concentration profiles for continuous ground-level point (Equation (16)), finite line (Equation (24)) and area (Equation (29)) sources of equal total source strength, so that $J_p = J_l l_y = J_a l_y l_x$. Similarly, in Fig. 7, we have presented concentration profiles for three area sources with dimensions $l_y \times l_x$ of 40×25 m^2, 80×50 m^2, and 160×100 m^2, respectively, and equivalent total source strengths ($J_p = J_a l_y l_x$). Figures 6 and 7 illustrate the importance of adequately representing the structure of the ground-level source, particularly when contaminant emissions are to be predicted in the vicinity of the source. As seen in Fig. 6, at sufficiently large downwind distances ($q_x \to \infty$), any finite line or area source can be accurately treated as a point source.

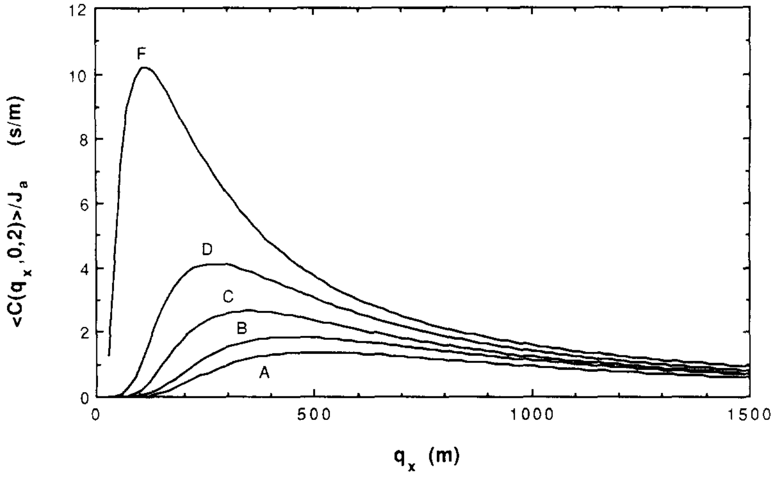

Fig. 4. Variation of normalized concentration with downwind distance and stability conditions ($q_z = 2$ m, $q_{z_0} = 0.1$ m, $v_d = 0$).

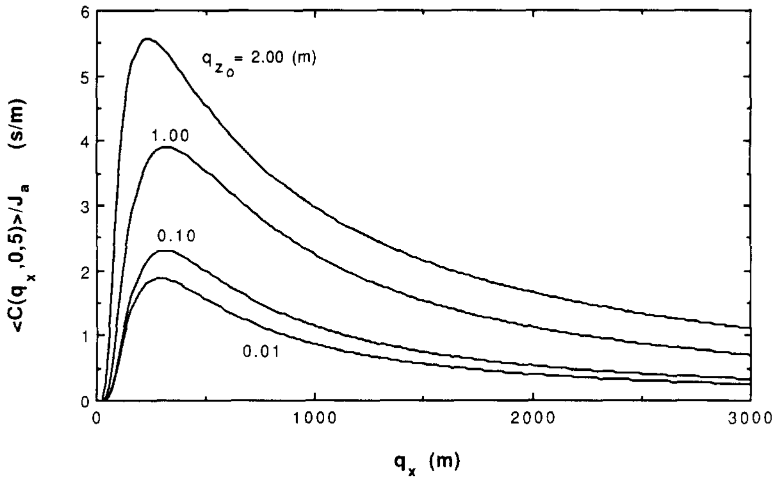

Fig. 5. Variation of normalized concentration with downwind distance and surface roughness length (F stability, $q_z = 5$ m, $v_d = 0$).

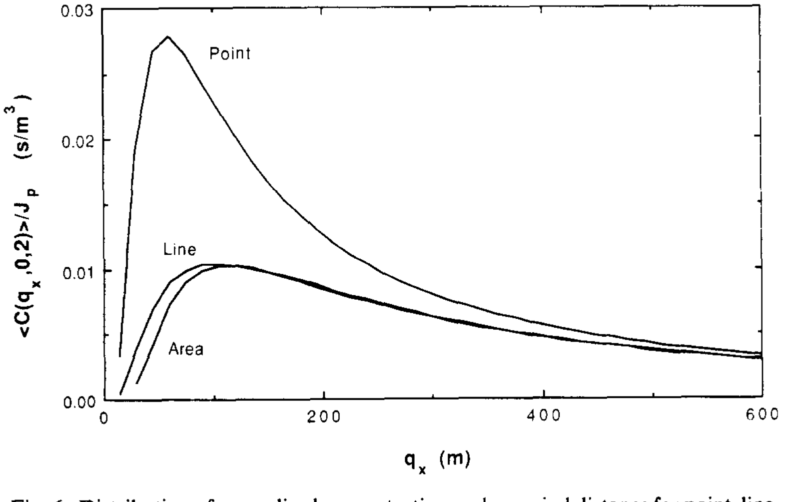

Fig. 6. Distribution of normalized concentration vs downwind distance for point, line and area sources of equal rate of total mass emitted ($J_p = J_l l_y = J_a l_y l_x$, F stability, $q_z = 2$ m, $q_{z_0} = 0.1$ m, $v_d = 0$).

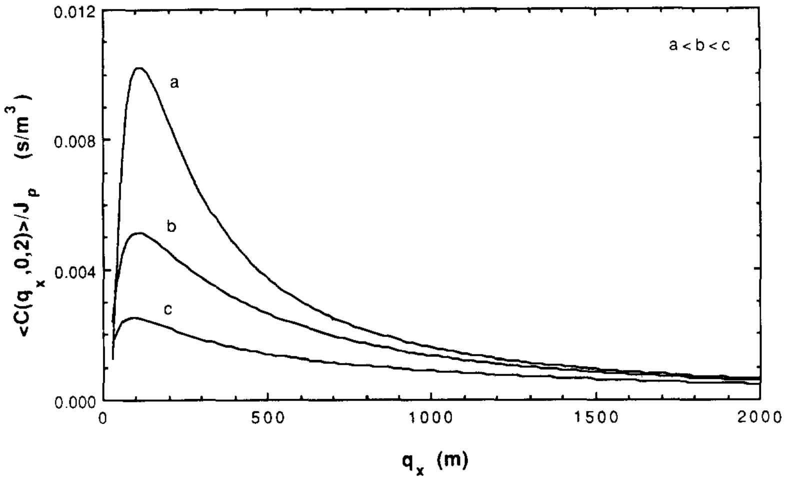

Fig. 7. Distribution of normalized concentration vs downwind distance for area sources of equal rate of total mass emitted and dimensions $l_y \times l_x$ (a) $40 \times 25\ m^2$, (b) 80 $\times 50\ m^2$, and (c) $160 \times 100\ m^2 (J_p = J_a l_y l_x$, F stability, $q_z = 2$ m, $q_{z_0} = 0.1$ m, $v_d = 0$).

SUMMARY AND CONCLUDING REMARKS

An analytical solution to a three-dimensional dispersion–deposition model for a continuous ground-level area source has been developed, and some of the features of the solution have been illustrated. The model assumes that contaminant emissions are non-buoyant and considers dry deposition as a sink mechanism.

Although the model presented has many advantages due to its analytical nature, some of the limitations inherent to the model are its inability: (a) to allow for non-steady contaminant emissions; (b) to incorporate arbitrary wind velocity and vertical eddy diffusivity profiles; and (c) to allow for wind direction changes with height above the surface.

Nonetheless, this model is applicable to a wide variety of ground surface roughness lengths at neutral or unstable atmospheric conditions. It is most useful for predictions of contaminant concentration near area sources where models assuming point or line sources may lead to overestimated concentration levels. Since the solution is analytical, it may also be useful for verifying the accuracy of numerical solutions to more comprehensive atmospheric dispersion models.

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